Published by Institute of Physics Publishing for SISSA

Received: *July 16, 2007*
Revised: *October 16, 2007*
Accepted: *October 23, 2007*
Published: *November 5, 2007*

Lovelock inflation and the number of large dimensions

Francesc Ferrer

CERCA, Department of Physics, Case Western Reserve University,
10900 Euclid Avenue, Cleveland, OH 44106-7079, U.S.A.
E-mail: francesc.ferrer@case.edu

Syksy Räsänen

CERN, Physics Department, Theory Unit,
CH-1211 Geneva 23, Switzerland
E-mail: syksy.rasanen@iki.fi

Abstract: We discuss an inflationary scenario based on Lovelock terms. These higher order curvature terms can lead to inflation when there are more than three spatial dimensions. Inflation will end if the extra dimensions are stabilised, so that at most three dimensions are free to expand. This relates graceful exit to the number of large dimensions.

Keywords: Classical Theories of Gravity, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Compactification and inflation. Perturbative string theory is most naturally formulated in 9+1 dimensions. The usual way of getting closer to the observed 3+1-dimensional universe is to compactify six spatial dimensions by hand so as to end up with four-dimensional Minkowski space times a six-dimensional compact manifold. (It is also possible to formulate critical string theory without extra spacetime dimensions; see e.g. the review [1].)

The extra dimensions are usually taken to be static (indeed, understanding of string theory in time-dependent backgrounds is still quite limited), and compactification is considered not to involve any dynamical evolution. In the search for a static split into large and small spatial dimensions, no explanation has emerged for why there should be three of the former and six of the latter. From the point of view of the ten-dimensional theory, there is no particular preference for six compact dimensions.

Even if the 3+6 split is taken for granted, there is a vast number of different ways of compactifying the six dimensions. Thus far, no unique, or even uniquely promising, compactification has emerged, and it has been suggested that there simply is no preferred way to compactify the extra dimensions. This could indicate a lack of predictivity in string theory (or that string theory is not correct), but it may rather show that some important principle is missing. There is no complete non-perturbative formulation of string theory, and it could be that the required ingredient is related to poorly understood non-perturbative aspects. A simpler possibility is that the split into three large and six small dimensions arises due to dynamical evolution, which is absent in the usual formulations of the problem, based as they are on a particle physics viewpoint with static manifolds, rather than a cosmological approach with evolving dimensions.

A somewhat analogous situation existed with respect to the puzzle of cosmological homogeneity and isotropy before the introduction of inflation. General relativity has a multitude of solutions, and though no rigorous measure in the space of solutions has been found, it would seem that the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) models are a subset of measure zero by any reasonable definition. So the question is: why is the universe, for a large segment of its evolution, well described by one of these very

JHEP11(2007)003

special solutions? Attempts to solve this problem in the context of general relativity [2] were unsuccessful until the introduction of inflation using ideas from particle physics [3]. From a modern viewpoint, the solution of the problem by accelerating expansion is intimately related to violation of the dominant energy condition ($\rho + 3p \geq 0$), an ingredient which may seem strange from a general relativistic point of view, but which is natural in particle physics. (However, it is not clear how generally inflation can start and lead to homogeneity and isotropy from an inhomogeneous and anisotropic initial state [4, 5].)

Perhaps taking into account the ingredient of dynamical evolution, which is natural from the cosmological point of view, could similarly be useful with the particle physics problem of compactification. At late times in the universe, the visible spatial dimensions expand, while any compact dimensions must be relatively static, so as not to conflict with the observational limits on the change of the gravitational coupling (see for example [6]). From the cosmological point of view, the question is: which mechanism is responsible for stabilising some of the dimensions while others are free to expand, and how does that mechanism determine the number of expanding dimensions?

Though compactification is a well-studied topic, relatively little work has been done on trying to explain why the number of large spatial dimensions should be three. The most notable exception is the study of string gas cosmology (SGC), where the dynamical determination of the number of large dimensions has been a central topic [7 – 9] (see [10, 11] for reviews). (There is also an alternative explanation for why we observe three large dimensions: that we live on a three-dimensional brane. There has been some work on trying to dynamically determine why three-branes would be preferred in this case [12, 13].)

In SGC, all spatial dimensions start on an equal footing, all compact and of the string size. The universe is filled with a hot gas of branes of all allowed dimensionalities. In the simplest versions of SGC the dimensions are toroidal, so that branes can wind around them, and resist expansion. (If the particle physics compactifications are unmotivated from a cosmological point of view, toroidal extra dimensions are in turn problematic for particle physics. See [14, 15] for discussion of more complex compactifications.) As the universe expands and cools down, winding and anti-winding modes annihilate, allowing further expansion. A simple counting argument suggests that p-branes and their anti-branes cannot find each other to annihilate in more than $2p + 1$ spatial dimensions, so at most $2p + 1$ dimensions can become large. For $p = 1$, corresponding to strings, this is three spatial dimensions. (Some quantitative studies of brane gases have cast doubt on this qualitative argument, see [16 – 21] for different analyses.)

Conceptually, inflation fits naturally into SGC: all dimensions are initially small, and inflation makes three of them macroscopically large. Instead of having separately inflation in the visible dimensions and static compactification in the extra dimensions, one could dynamically explain decompactification via inflation. (This idea was introduced in an earlier Kaluza-Klein context in [22].) However, the practical implementation of inflation in SGC is problematic, since inflation dilutes the string gas which stabilises the extra dimensions, and no compelling inflationary scenario in SGC has been found [23 – 29]. (For alternatives to inflation in SGC, see [30 – 33].) An extra ingredient is needed, something that stabilises the extra dimensions even against inflation. We will point out that if such

JHEP11(2007)003

a mechanism is found, stabilising the extra dimensions may be directly related to ending inflation in the visible dimensions.

Lovelock gravity. We are interested in inflation in a higher-dimensional space. In a general metric theory of gravity in d dimensions, the equation of motion sets the energy-momentum tensor equal to some covariantly conserved rank two tensor built from the metric and its derivatives. Demanding the equations of motion to be of second order [34–36] strongly constrains the terms which can appear. In four dimensions, there are only two local tensors with the required properties: the Einstein tensor, and the metric itself, the latter corresponding to the cosmological constant [37, 38].

In more than four dimensions, the Einstein tensor is no longer the unique covariantly conserved non-trivial tensor constructed from the metric and its first and second derivatives. In d dimensions there are exactly $[d/2]$ ($d/2$ rounded up) such symmetric tensors (and corresponding local Lagrange densities), known as the Lovelock tensors [37]. (The Einstein tensor is still the only covariantly conserved local tensor which is linear in second derivatives.)

The approach which leads to Einstein gravity in four dimensions gives Lovelock gravity in higher dimensions. The first new contribution to the Lagrange density, quadratic in curvature, is the well-known Gauss-Bonnet term. In four dimensions it reduces to a topological quantity and does not contribute to the equations of motion. (The higher Lovelock terms vanish in four dimensions.)

From the viewpoint of string theory, the Lovelock Lagrangians may be said to be preferred, as they lead to a unitary and ghost-free low energy effective theory [39, 40]. However, since the effective theory is defined only up to field redefinitions, Lovelock Lagrangians should be (at least to second order in the Riemann tensor) physically equivalent to non-Lovelock Lagrangians [41]. This means that the seeming problems of non-Lovelock terms are expected to become apparent only at large curvatures, where the effective theory does not apply.

We do not consider the details of the string theory context, and will simply look at ten-dimensional cosmology with Lovelock gravity. From the string theory point of view, we are ignoring the extra fields present in addition to the metric; in particular we are assuming that the dilaton has been stabilised in a manner that does not impose any constraints on the metric. We find that Lovelock gravity can naturally involve inflation in higher dimensions. Furthermore, the end of inflation is tied up with the stabilisation of the hidden dimensions: if the extra dimensions are kept small, the universe soon becomes effectively four-dimensional. This will in turn end inflation in the visible dimensions, because the contribution of the Lovelock terms vanishes in four dimensions: graceful exit from inflation is tied to (at most) three spatial dimensions becoming large.

In section 2 we describe Lovelock gravity, explain the inflationary mechanism and point out the connection between graceful exit and stabilisation. We briefly discuss some ideas for ending inflation and summarise in section 3.

JHEP11(2007)003

2. Lovelock inflation

The action and the equation of motion. In a metric theory of gravity in d dimensions, the most general local Lagrange density which leads to equations of motion containing at most second order derivatives of the metric is [37]

$$
\begin{aligned}
L_{\rm love} &= \sum_{n=0}^{[d/2]} c_n L_n \\
&\equiv \sum_{n=0}^{[d/2]} c_n 2^{-n} \delta^{\alpha_1 \cdots \alpha_{2n}}_{\beta_1 \cdots \beta_{2n}} R_{\alpha_1 \alpha_2}{}^{\beta_1 \beta_2} \ldots R_{\alpha_{2n-1} \alpha_{2n}}{}^{\beta_{2n-1} \beta_{2n}} \, ,
\end{aligned}
\tag{2.1}
$$

where $\delta^{\alpha_1 \cdots \alpha_k}_{\beta_1 \cdots \beta_k}$ is the generalised Kronecker delta symbol (totally antisymmetric in both upper and lower indices), $[d/2]$ is $d/2$ rounded up to the nearest integer and c_n are constants; by definition $L_0 \equiv 1$. The first term is the cosmological constant, the second is the Einstein-Hilbert Lagrange density and the third is the Gauss-Bonnet Lagrange density. We will consider the case $d = 10$, but for simplicity we drop the terms or order three and four in the Riemann tensor; including them is straightforward. The action is

$$
\begin{aligned}
S_{\rm love} &= \int \mathrm{d}^{10}x \sqrt{-g}\, (c_0 L_0 + c_1 L_1 + c_2 L_2) + S_{\rm m} \\
&= \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \sqrt{-g} \left[-2\Lambda + R + \alpha(R^2 - 4 R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}) \right] \\
&\quad + \int \mathrm{d}^{10}x \sqrt{-g} L_{\rm m} \, ,
\end{aligned}
\tag{2.2}
$$

where $L_{\rm m}$ is the Lagrangian of the matter fields present and we have denoted $c_0 = -\Lambda/\kappa^2$, $c_1 = 1/(2\kappa^2)$ and $c_2 = \alpha/(2\kappa^2)$, where Λ is the cosmological constant, κ^2 is the 10-dimensional gravitational coupling and α is the Gauss-Bonnet coefficient.

The equation of motion following from (2.2) is

$$
\kappa^2 T_{\mu\nu} = G_{\mu\nu} + \alpha H_{\mu\nu} \, , \tag{2.3}
$$

where κ^2 is the gravitational coupling in d dimensions, $T_{\mu\nu}$ is the energy-momentum tensor (which we take to include the cosmological constant), $G_{\mu\nu}$ is the Einstein tensor and $H_{\mu\nu}$ is the Gauss-Bonnet tensor given by

$$
\begin{aligned}
H_{\mu\nu} &= 2 R R_{\mu\nu} - 4 R_{\mu\alpha} R^{\alpha}{}_{\nu} - 4 R_{\alpha\beta} R^{\alpha}{}_{\mu}{}^{\beta}{}_{\nu} + 2 R_{\mu\alpha\beta\gamma} R_{\nu}{}^{\alpha\beta\gamma} \\
&\quad - \frac{1}{2} g_{\mu\nu} \left(R^2 - 4 R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \right) \, .
\end{aligned}
\tag{2.4}
$$

The metric. We take the metric to be the simplest generalisation of the spatially flat Friedmann-Robertson-Walker (FRW) universe, homogeneous and separately isotropic in the visible and the extra dimensions:

$$
\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \sum_{i=1}^{3} \mathrm{d}x^i \mathrm{d}x^i + b(t)^2 \sum_{j=1}^{6} \mathrm{d}y^j \mathrm{d}y^j \, , \tag{2.5}
$$

where x^i and y^j are the spatial coordinates in the visible and extra dimensions, respectively.

Given the symmetries of the metric (2.5), the energy-momentum tensor is

$$T^\mu_{\ \nu} = \mathrm{diag}(-\rho(t), p(t), p(t), p(t), P(t), P(t), P(t), P(t), P(t), P(t)) \ . \tag{2.6}$$

With (2.5) and (2.6), the equation of motion (2.3) reads

$$\kappa^2\rho = 3\frac{\dot{a}^2}{a^2} + 18\frac{\dot{a}}{a}\frac{\dot{b}}{b} + 15\frac{\dot{b}^2}{b^2} + 36\alpha\frac{\dot{b}}{b}\left(2\frac{\dot{a}^3}{a^3} + 15\frac{\dot{a}^2}{a^2}\frac{\dot{b}}{b} + 20\frac{\dot{a}}{a}\frac{\dot{b}^2}{b^2} + 5\frac{\dot{b}^3}{b^3}\right) \tag{2.7}$$

$$\begin{aligned}\kappa^2 p = &-\left(2\frac{\ddot{a}}{a} + 6\frac{\ddot{b}}{b} + \frac{\dot{a}^2}{a^2} + 12\frac{\dot{a}}{a}\frac{\dot{b}}{b} + 15\frac{\dot{b}^2}{b^2}\right) \\ &-12\alpha\left(4\frac{\dot{a}}{a}\frac{\dot{b}}{b}\frac{\ddot{a}}{a}+10\frac{\dot{b}^2}{b^2}\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}\frac{\ddot{b}}{b}+20\frac{\dot{a}}{a}\frac{\dot{b}}{b}\frac{\ddot{b}}{b}+20\frac{\dot{b}^2}{b^2}\frac{\ddot{b}}{b}+15\frac{\dot{a}^2}{a^2}\frac{\dot{b}^2}{b^2}+40\frac{\dot{a}}{a}\frac{\dot{b}^3}{b^3}+15\frac{\dot{b}^4}{b^4}\right)\end{aligned} \tag{2.8}$$

$$\begin{aligned}\kappa^2(\rho-3p+2P) = &\ 8\frac{\ddot{b}}{b}+24\frac{\dot{a}}{a}\frac{\dot{b}}{b}+40\frac{\dot{b}^2}{b^2}+24\alpha\left(-\frac{\dot{a}^2}{a^2}\frac{\ddot{a}}{a}-4\frac{\dot{a}}{a}\frac{\dot{b}}{b}\frac{\ddot{a}}{a}+5\frac{\dot{b}^2}{b^2}\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}\frac{\ddot{b}}{b}+10\frac{\dot{a}}{a}\frac{\dot{b}}{b}\frac{\ddot{b}}{b}\right. \\ &\left.+20\frac{\dot{b}^2}{b^2}\frac{\ddot{b}}{b}-2\frac{\dot{a}^3}{a^3}\frac{\dot{b}}{b}+15\frac{\dot{a}^2}{a^2}\frac{\dot{b}^2}{b^2}+60\frac{\dot{a}}{a}\frac{\dot{b}^3}{b^3}+25\frac{\dot{b}^4}{b^4}\right) \ .\end{aligned} \tag{2.9}$$

As in the usual FRW case, not all of the equations are independent, and (as long as $\dot{b} \neq 0$) we can simply use (2.7) and (2.8) along with the conservation law of the energy-momentum tensor:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) + 6\frac{\dot{b}}{b}(\rho + P) = 0 \ . \tag{2.10}$$

When the extra dimensions are static, $\dot{b} = 0$, the components of the Gauss-Bonnet tensor in the four visible directions vanish, and we recover the usual FRW equations in the visible directions. This is expected, since in four dimensions the Gauss-Bonnet term does not contribute to the equations of motion. Note that the components of the Gauss-Bonnet tensor in the direction of the extra dimensions do not vanish, though its contribution is negligible at low curvatures. The higher order Lovelock tensors vanish when $\dot{b} = 0$ (the expressions for them can be found in [42]), so if we used them instead of the Gauss-Bonnet term, the dynamics would completely reduce to the FRW case when the extra dimensions are stabilised. This is presumably related to the fact that in four dimensions the Gauss-Bonnet action is total derivative, while the higher order Lovelock actions are identically zero. For discussion of cosmology with Lovelock terms, see [42 − 45].

Inflation. Let us first look at the case when there is no distinction between the visible and extra dimensions, so the universe is isotropic, $a = b$. Then (2.7)–(2.10) reduce to

$$36H^2 + 1512\alpha H^4 = \kappa^2\rho \tag{2.11}$$

$$\dot{\rho} + 9H(\rho + p) = 0 \, , \tag{2.12}$$

where $H \equiv \dot{a}/a$. The conservation law of the energy-momentum tensor (2.12) is the usual one. But the Hubble law has qualitatively new features if $\alpha < 0$ (which we assume from now on). (For string theory, the second order coefficient α is, to leading order, zero for superstrings, and positive for heterotic string theory. However, this is not the case for all higher order Lovelock terms [46, 47].)

The Hubble law (2.11) is plotted in figure 1, along with the usual FRW Hubble law for comparison. The Gauss-Bonnet Hubble law has two branches, with different vacua and different dynamics. On branch I the vacuum is Minkowski space, whereas on branch II the vacuum is de Sitter space with Hubble parameter $H = 1/\sqrt{42|\alpha|}$. The vacua have been analysed in [48 − 50]. In the de Sitter vacuum, the gravitational excitations are ghosts, implying that it is not a stable solution.

On branch I, the behaviour is the usual FRW one at low energies ($\kappa^2\rho \ll 1/|\alpha|$), with modifications at high energies. For matter satisfying $\rho + p > 0$, the Hubble parameter decreases. In contrast, on branch II the universe undergoes superinflation ($\dot{H} > 0$) if the matter obeys $\rho + p > 0$: the smaller the energy density, the faster the expansion of the universe. Likewise, a positive cosmological constant decreases the expansion rate, instead of increasing it.

On both branches, the energy density and all other observables are non-divergent at all times: upon approaching what would be a curvature singularity in the FRW case, the energy density levels off. The usual singularity theorems of general relativity do not apply to Gauss-Bonnet gravity, so it would be possible for the spacetime to be non-singular. (If the Gauss-Bonnet tensor is considered as an effective energy-momentum tensor, it violates the null energy condition.) However, even though there is no curvature singularity, the spacetime is geodesically incomplete and thus singular ([51], page 212). An easy way to see this is to consider a collapsing universe on branch I: as the energy density increases to the value at the peak, $\kappa^2\rho = 3/(14|\alpha|)$, the universe cannot collapse further and simply ceases to exist.

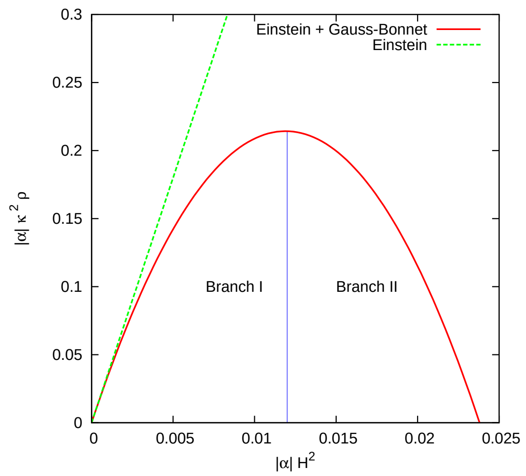

Figure 1: The Hubble law with the Gauss-Bonnet term, and the Einstein Hubble law for comparison.

That the Gauss-Bonnet term leads to an inflationary solution when $\alpha < 0$ can be understood in the following way. If a tensor does not contain higher than second order derivatives and is covariantly conserved, its 00-component cannot contain higher than first order derivatives. (Considering the Gauss-Bonnet tensor as an effective energy-momentum tensor, one can see this from the conservation law (2.12): if ρ had second order derivatives, p would be third order.) Given a tensor with dimension m^4 and assuming the spatially flat FRW metric, the 00-component is then proportional to H^4, the only available quantity of the correct dimension. This leads immediately to the Hubble law (2.11); only the coefficients 36 and 1512 depend on the detailed structure of the Gauss-Bonnet tensor (and are specific to 10 dimensions).

The structure is the same for all dimensions $d > 4$ where the Gauss-Bonnet tensor is non-zero. If one includes all the higher order Lovelock tensors, the Hubble law (2.11) becomes of order [d/2]-1 with respect to H^2. For $d = 10$, there are three Lovelock terms (in addition to the cosmological constant and the Einstein tensor term), so the Hubble law is quartic in H^2. As long as the Hubble law has at least one non-zero solution for $\rho = 0$, there is at least one inflationary branch.

As an aside, we note that this structure can be realised even in four dimensions. If the metric is conformally flat, there exist additional $d/2$ (rounded down) tensors of increasing dimensionality which are second order in derivatives of the metric and covariantly conserved [52]. The tensor which has dimension m^4 is usually labeled $H^{(3)}_{\mu\nu}$ ([53], page 183). Including this tensor and taking the FRW metric leads to a Hubble law of the form (2.11). Some of the properties of the modified Hubble law discussed above have been earlier mentioned in this four-dimensional context [54]. The tensor $H^{(3)}_{\mu\nu}$ can even be extended to first order in perturbation theory around the FRW background [55]. Lovelock's theorem guarantees that there is no local extension of the tensor $H^{(3)}_{\mu\nu}$ to four-dimensional general spacetimes, but there is a non-local extension (which is no longer second order in the derivatives), connected with the trace anomaly [56].

Graceful exit. In order for the inflationary mechanism to be cosmologically relevant, there has to be a way of ending inflation –in our case, getting from branch II to branch I– as well as sorting out only three spatial dimensions to inflate. In fact, the problems of ending inflation and breaking isotropy are related. The Hubble law (2.7) in the general case with $a \neq b$ reads

$$3(1+6\lambda+5\lambda^2)H^2 - 36\lambda(2+15\lambda+20\lambda^2+5\lambda^3)|\alpha|H^4 = \kappa^2\rho \,, \tag{2.13}$$

where $H \equiv \dot{a}/a$, and $\lambda(t) \equiv (\dot{b}/b)/H$. If the evolution given by the Hubble law and (2.9), (2.10) is such that $\lambda \to 0$, the extra dimensions stabilise, the Hubble parabola straightens out and branch II disappears. In other words, inflation ends and the standard Hubble law is recovered. This happens only if the number of dimensions which are left free to expand is at most three. For p large spatial dimensions and $9-p$ extra dimensions, the Hubble law would be

$$\begin{aligned}&\left[\frac{1}{2}p(p-1)+p(9-p)\lambda+\frac{1}{2}(9-p)(8-p)\lambda^2\right]H^2-\left[\frac{1}{2}p(p-1)(p-2)(p-3)+2p(9-p)(p-1)(p-2)\lambda\right.\\&+3p(p-1)(9-p)(8-p)\lambda^2+2p(9-p)(8-p)(7-p)\lambda^3\\&\left.+\frac{1}{2}(9-p)(8-p)(7-p)(6-p)\lambda^4\right]|\alpha|H^4=\kappa^2\rho\,.\end{aligned} \tag{2.14}$$

If the extra dimensions are stabilised, $\lambda = 0$, we obtain

$$\frac{1}{2}p(p-1)H^2 - \frac{1}{2}p(p-1)(p-2)(p-3)|\alpha|H^4 = \kappa^2\rho \,. \tag{2.15}$$

It is transparent that inflation persists unless the number of large dimensions is at most three. Note that, for non-zero ρ, stabilisation is not consistent with zero or one large dimensions. However, there is no obvious obstruction to having two large dimensions instead of three. This is a constraint on inflation in the visible dimensions, assuming that the extra dimensions stabilise. (If only higher order Lovelock terms were present instead of the Gauss-Bonnet term, the number of preferred dimensions would be larger. For the third order Lovelock term, inflation would end for five or less spatial dimensions, and the fourth order term would yield seven or less.)

So, if there is a mechanism which allows only a three-dimensional subspace to become large and slows down expansion of the other dimensions, the universe will become effectively four-dimensional and the contribution of the Gauss-Bonnet tensor in the visible directions will go to zero, ending inflation. Finding such a mechanism was the original aim of SGC [8]. It is not clear whether three dimensions are dynamically preferred or not [16 − 21]. But even if three dimensions are selected in a slowly expanding space, with the extra dimensions stabilised by a gas of strings, in an inflating space the string gas will be diluted and the extra dimensions will be free to expand [6, 24]. Such a destabilising effect is present even in a matter-dominated universe, though in that case the string gas can counter the effect and rein in the extra dimensions [6].

We studied whether this stabilisation mechanism with a gas of strings or higher-dimensional branes could end Lovelock inflation. We used the energy-momentum tensor for the string gas given in [6], and its generalisation for higher-dimensional branes. While strings indeed slow down the expansion initially, their effect is soon diluted to negligible levels by inflation. Since the energy density of higher-dimensional branes is diluted more slowly, they could potentially have a stronger impact. However, the behaviour is essentially the same: the brane gas does slow down the expansion of the extra dimensions, but the effect is too weak, and space isotropizes, with all dimensions growing large.

So, while we have connected the end of inflation with (at most) three spatial dimensions becoming large, we have not managed to explain why the other dimensions would be stabilised. In the next section, we will discuss some ideas towards ending inflation and getting from the inflationary branch to the FRW branch.

3. Discussion

Ending and starting inflation. The line of reasoning leading to Lovelock gravity (writing down all terms consistent with second order equations of motion) is motivated for a classical theory. However, it may be inadequate when quantum effects are included, because anomalies can break classical symmetries, leading to a modification of the low energy action. In the case of quantum fields coupled to classical gravity, the trace anomaly leads to terms higher than second order in the derivatives, and one can argue that they should be included in the effective action of gravity [53, 56]. The terms related to the trace anomaly were used in the first inflationary model [57]. It would be interesting to investigate their impact on Lovelock inflation. In particular, the trace anomaly terms could destabilise the de Sitter solution and lead to a graceful exit, like in [57]. Like the Lovelock terms, the trace anomaly is sensitive to the number of dimensions, though it is not clear that it would prefer three large dimensions over some other number.

JHEP11(2007)003

From the string theory point of view, the most conspicuous missing ingredient is the dilaton. We have simply assumed that the dilaton is stabilised in a way which does not impose constraints on the Einstein equation. In general, if we include the dilaton in the action, we have in addition to the Einstein equation the dilaton equation of motion. Taking the dilaton to be constant then leads to a constraint equation for the metric. In the present context with the Lovelock terms, the constraint removes the de Sitter solution, leaving only the Minkowski vacuum (somewhat like in the inflationary scenario of [58]). This might work well, since it means that any period of inflation would be transient, and the dilaton could serve to end inflation and take the universe to the FRW branch. However, while this would tie the end of inflation with dilaton stabilisation, there is no apparent connection to having three large spatial dimensions.

Apart from the trace anomaly or dilaton, the fact that the gravitational excitations around the de Sitter solution are ghosts implies that it is unstable [48, 49]. Such an instability could also provide a satisfactory transition to the FRW branch.

One advantage of Lovelock inflation is that it is not inconsistent with a mechanism that would solve the cosmological constant problem by cancelling the gravitational effects of vacuum energy, unlike usual scalar field models of inflation [59]. (For an inflationary mechanism which is instead based on this kind of a cancellation mechanism, see [60].)

Another problem of conventional scalar field models is getting inflation started. Unless the null energy condition is violated, starting inflation requires homogeneity over at least a Hubble-sized patch [4]. As we have noted, the Lovelock tensors (considered as an effective source) violate the null energy condition, so there is no obstruction, in principle, to inflation starting in an inhomogeneous patch and creating homogeneity, rather than simply amplifying it. Studies of inhomogeneous spacetimes would be needed to establish how this works quantitatively; the issue is not fully worked out even in the usual inflationary case [5].

Conclusion. In the usual formulation of string theory, six spatial dimensions are compactified by hand, whereas three are taken to be large. Since the most successful scenario of the early universe, inflation, produces exponentially large dimensions starting from small ones, it seems elegant to combine inflation and the question of why some dimensions are much larger than others. In this framework, all dimensions would start at some small natural scale, and inflation would explain why three of them inflate to become macroscopically large.

We have discussed how a natural generalisation of Einstein gravity in higher dimensions, Lovelock gravity, can give inflationary solutions. The inflation will end if one stabilises the extra dimensions, since the non-Einstein Lovelock terms do not contribute in 3+1 dimensions or less. This ties the graceful exit problem of inflation to the number of spatial dimensions: Lovelock inflation will only end if the number of large spatial dimensions becomes three or less.

String gas cosmology supplies a mechanism for selecting only three dimensions to expand. However, while this mechanism works during both the radiation- and matter-dominated eras, it fails for inflation. Taking into account the trace anomaly or the dilaton could lead to a viable graceful exit, but it is not clear whether the number of large spatial dimensions would emerge correctly. Further work is needed on stabilising extra dimensions: what we have shown is that the solution of the stabilisation issue may be directly relevant for inflation.

Acknowledgments

SR thanks Antonio Padilla for discussions, Kari Enqvist and Esko Keski-Vakkuri for discussions in the early stages of this work and the Helsinki Institute of Physics for hospitality. FF is supported in part by grants from the DOE and NSF at Case Western Reserve University.

References

[1] G.T. Horowitz, *Spacetime in string theory*, *New J. Phys.* **7** (2005) 201 [gr-qc/0410049].

[2] C.W. Misner, *The isotropy of the universe*, *Astrophys. J.* **151** (1968) 431; *Mixmaster universe*, *Phys. Rev. Lett.* **22** (1969) 1071.

[3] A.H. Guth, *The inflationary universe: a possible solution to the horizon and flatness problems*, *Phys. Rev.* **D 23** (1981) 347.

[4] T. Vachaspati and M. Trodden, *Causality and cosmic inflation*, *Phys. Rev.* **D 61** (2000) 023502 [gr-qc/9811037]; *What is the homogeneity of our universe telling us?*, *Mod. Phys. Lett.* **A 14** (1999) 1661 [gr-qc/9905091].

[5] G.F.R. Ellis, 83 *years of general relativity and cosmology: progress and problems*, *Class. and Quant. Grav.* **16** (1999) A37; *Relativistic cosmology* 1999*: issues and problems*, *Gen. Rel. Grav.* **32** (2000) 1135.

[6] F. Ferrer and S. Räsänen, *Dark energy and decompactification in string gas cosmology*, *JHEP* **02** (2006) 016 [hep-th/0509225].

[7] J. Kripfganz and H. Perlt, *Cosmological impact of winding strings*, *Class. and Quant. Grav.* **5** (1988) 453.

[8] R.H. Brandenberger and C. Vafa, *Superstrings in the early universe*, *Nucl. Phys.* **B 316** (1989) 391.

[9] A.A. Tseytlin and C. Vafa, *Elements of string cosmology*, *Nucl. Phys.* **B 372** (1992) 443 [hep-th/9109048].

[10] T. Battefeld and S. Watson, *String gas cosmology*, *Rev. Mod. Phys.* **78** (2006) 435 [hep-th/0510022].

[11] R.H. Brandenberger, *Conceptual problems of inflationary cosmology and a new approach to cosmological structure formation*, hep-th/0701111.

[12] R. Durrer, M. Kunz and M. Sakellariadou, *Why do we live in* $3+1$ *dimensions?*, *Phys. Lett.* **B 614** (2005) 125 [hep-th/0501163];
M. Sakellariadou, *The origin of spacetime dimensionality*, hep-th/0701024.

[13] A. Karch and L. Randall, *Relaxing to three dimensions*, *Phys. Rev. Lett.* **95** (2005) 161601 [hep-th/0506053].

[14] D.A. Easson, *Brane gases on $K3$ and Calabi-Yau manifolds*, *Int. J. Mod. Phys.* **A 18** (2003) 4295 [hep-th/0110225].

[15] R. Easther, B.R. Greene and M.G. Jackson, *Cosmological string gas on orbifolds*, *Phys. Rev.* **D 66** (2002) 023502 [hep-th/0204099].

[16] M. Sakellariadou, *Numerical experiments in string cosmology*, *Nucl. Phys.* **B 468** (1996) 319 [hep-th/9511075].

[17] R. Easther, B.R. Greene, M.G. Jackson and D. Kabat, *Brane gas cosmology in M-theory: late time behavior*, *Phys. Rev.* **D 67** (2003) 123501 [hep-th/0211124].

[18] B.A. Bassett, M. Borunda, M. Serone and S. Tsujikawa, *Aspects of string-gas cosmology at finite temperature*, *Phys. Rev.* **D 67** (2003) 123506 [hep-th/0301180].

[19] R. Easther, B.R. Greene, M.G. Jackson and D. Kabat, *Brane gases in the early universe: thermodynamics and cosmology*, *JCAP* **01** (2004) 006 [hep-th/0307233].

[20] R. Easther, B.R. Greene, M.G. Jackson and D. Kabat, *String windings in the early universe*, *JCAP* **02** (2005) 009 [hep-th/0409121].

[21] R. Danos, A.R. Frey and A. Mazumdar, *Interaction rates in string gas cosmology*, *Phys. Rev.* **D 70** (2004) 106010 [hep-th/0409162].

[22] Q. Shafi and C. Wetterich, *Cosmology from higher dimensional gravity*, *Phys. Lett.* **B 129** (1983) 387; *Inflation with higher dimensional gravity*, *Phys. Lett.* **B 152** (1985) 51; *Inflation from higher dimensions*, *Nucl. Phys.* **B 289** (1987) 787.

[23] R. Brandenberger, D.A. Easson and A. Mazumdar, *Inflation and brane gases*, *Phys. Rev.* **D 69** (2004) 083502 [hep-th/0307043].

[24] S.P. Patil and R. Brandenberger, *Radion stabilization by stringy effects in general relativity and dilaton gravity*, *Phys. Rev.* **D 71** (2005) 103522 [hep-th/0401037].

[25] A. Kaya, *Volume stabilization and acceleration in brane gas cosmology*, *JCAP* **08** (2004) 014 [hep-th/0405099].

[26] T. Biswas, R. Brandenberger, D.A. Easson and A. Mazumdar, *Coupled inflation and brane gases*, *Phys. Rev.* **D 71** (2005) 083514 [hep-th/0501194].

[27] D.A. Easson and M. Trodden, *Moduli stabilization and inflation using wrapped branes*, *Phys. Rev.* **D 72** (2005) 026002 [hep-th/0505098].

[28] N. Shuhmaher and R. Brandenberger, *Brane gas-driven bulk expansion as a precursor stage to brane inflation*, *Phys. Rev. Lett.* **96** (2006) 161301 [hep-th/0512056].

[29] T. Battefeld and N. Shuhmaher, *Predictions of dynamically emerging brane inflation models*, *Phys. Rev.* **D 74** (2006) 123501 [hep-th/0607061].

[30] A. Nayeri, R.H. Brandenberger and C. Vafa, *Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology*, *Phys. Rev. Lett.* **97** (2006) 021302 [hep-th/0511140].

[31] R.H. Brandenberger and N. Shuhmaher, *The confining heterotic brane gas: a non-inflationary solution to the entropy and horizon problems of standard cosmology*, *JHEP* **01** (2006) 074 [hep-th/0511299]

[32] R.H. Brandenberger et al, *More on the spectrum of perturbations in string gas cosmology*, *JCAP* **11** (2006) 009 [hep-th/0608186].

JHEP11(2007)003

[33] N. Kaloper, L. Kofman, A. Linde and V. Mukhanov, *On the new string theory inspired mechanism of generation of cosmological perturbations*, *JCAP* **10** (2006) 006 [hep-th/0608200].

[34] F. Müller-Hoissen, *Higher derivative versus second order field equations*, *Annalen Phys.* **48** (1991) 543.

[35] J.Z. Simon, *Higher derivative Lagrangians, nonlocality, problems and solutions*, *Phys. Rev.* **D 41** (1990) 3720.

[36] R.P. Woodard, *Avoiding dark energy with $1/R$ modifications of gravity*, astro-ph/0601672.

[37] D. Lovelock, *The Einstein tensor and its generalizations*, *J. Math. Phys.* **12** (1971) 498.

[38] D. Lovelock, *The four-dimensionality of space and the Einstein tensor*, *J. Math. Phys.* **13** (1972) 874.

[39] B. Zwiebach, *Curvature squared terms and string theories*, *Phys. Lett.* **B 156** (1985) 315.

[40] B. Zumino, *Gravity theories in more than four-dimensions*, *Phys. Rept.* **137** (1986) 109.

[41] S. Deser and A.N. Redlich, *String induced gravity and ghost freedom*, *Phys. Lett.* **B 176** (1986) 350 [*Erratum ibid.* **B 186** (1987) 461].

[42] J. Demaret, H. Caprasse, A. Moussiaux, P. Tombal and D. Papadopoulos, *Ten-dimensional Lovelock-type space-times*, *Phys. Rev.* **D 41** (1990) 1163.

[43] F. Müller-Hoissen, *Dimensionally continued Euler forms, Kaluza-Klein cosmology and dimensional reduction*, *Class. and Quant. Grav.* **3** (1986) 665.

[44] J. Kripfganz and H. Perlt, *Higher dimensional cosmology and the Gauss-Bonnet term*, *Acta Phys. Polon.* **B18** (1987) 997.

[45] N. Deruelle and L. Farina-Busto, *The Lovelock gravitational field equations in cosmology*, *Phys. Rev.* **D 41** (1990) 3696.

[46] A.A. Tseytlin, *Heterotic - type I superstring duality and low-energy effective actions*, *Nucl. Phys.* **B 467** (1996) 383 [hep-th/9512081].

[47] K.-i. Maeda and N. Ohta, *Inflation from superstring and M-theory compactification with higher order corrections I*, *Phys. Rev.* **D 71** (2005) 063520 [hep-th/0411093].

[48] D.G. Boulware and S. Deser, *String-generated gravity models*, *Phys. Rev. Lett.* **55** (1985) 2656.

[49] R.C. Myers and J.Z. Simon, *Black hole thermodynamics in Lovelock gravity*, *Phys. Rev.* **D 38** (1988) 2434.

[50] B. Whitt, *Spherically symmetric solutions of general second order gravity*, *Phys. Rev.* **D 38** (1988) 3000.

[51] R.M. Wald, *General relativity*, Chicago University Press, Chicago U.S.A. (1984).

[52] K.A. Meissner and M. Olechowski, *Domain walls without cosmological constant in higher order gravity*, *Phys. Rev. Lett.* **86** (2001) 3708 [hep-th/0009122].

[53] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, Cambridge U.K. (1982).

[54] R.M. Wald, *Axiomatic renormalization of the stress tensor of a conformally invariant field in conformally flat space-times*, *Ann. Phys. (NY)* **110** (1978) 472.

[55] G.T. Horowitz and R.M. Wald, *Quantum stress energy in nearly conformally flat space-times*, *Phys. Rev.* **D 21** (1980) 1462; *Quantum stress energy in nearly conformally flat space-times. II. Correction of formula*, *Phys. Rev.* **D 25** (1982) 3408.

[56] P.O. Mazur and E. Mottola, *Weyl cohomology and the effective action for conformal anomalies*, *Phys. Rev.* **D 64** (2001) 104022 [hep-th/0106151].

[57] A.A. Starobinsky, *A new type of isotropic cosmological models without singularity*, *Phys. Lett.* **B 91** (1980) 99.

[58] J.R. Ellis, N. Kaloper, K.A. Olive and J. Yokoyama, *Topological R^4 inflation*, *Phys. Rev.* **D 59** (1999) 103503 [hep-ph/9807482].

[59] R.H. Brandenberger, *Inflation and the theory of cosmological perturbations*, astro-ph/9711106.

[60] N.C. Tsamis and R.P. Woodard, *Quantum gravity slows inflation*, *Nucl. Phys.* **B 474** (1996) 235 [hep-ph/9602315]; *The quantum gravitational back-reaction on inflation*, *Ann. Phys. (NY)* **253** (1997) 1 [hep-ph/9602316];
L.R. Abramo, N.C. Tsamis and R.P. Woodard, *Cosmological density perturbations from a quantum gravitational model of inflation*, *Fortschr. Phys.* **47** (1999) 389 [astro-ph/9803172];
R.P. Woodard, *Effective field equations of the quantum gravitational back-reaction on inflation* astro-ph/0111462.